Harder Questions

1. If $y = \frac{u}{v}$ and $\frac{dy}{dx} = \frac{p}{q}$ where p, q, u, v are functions of x show that at a stationary point $\frac{d^2y}{dx^2}$ has the same sign as $\frac{dp}{dx}$. How could this result be used?

2. If $f(x) = x^3 + 3x^2 - 9x - 21$ show that

   (a) the stationary points are at $(-3, 6), (1, -26)$

   (b) $f''(-3) = -12$ and $f''(1) = 12$

   (c) the point of inflexion is at the midpoint of the 2 stationary points

3. Generalise the results of question 2 to all cubics by showing:

   (a) if $3ax^2 + 2bx + c = 0$ has roots α and β then

      i. $\alpha + \beta = -\frac{2b}{3a}$

      ii. $\alpha^2 + \beta^2 = \frac{4b^2 - 6ac}{9a}$

      iii. $\alpha^3 + \beta^3 = \frac{-8b^3 + 18abc}{27a^3}$

   (b) If $f(x) = ax^3 + bx^2 + cx + d$ and $x = \alpha$ and $x = \beta$ are the x-coordinates of its stationary points show that $f''(\alpha) + f''(\beta) = 0$ and hence $f''(\alpha) = -f''(\beta)$

   (c) * If $f''(x) = 0$ show that

      i. $x = -\frac{b}{2a}$

      ii. $f(x) = \frac{2b^3 - 9abc + 27a^2d}{27a^2}$

      iii. $f(\alpha) + f(\beta) = \frac{4b^3 - 18abc + 54a^2d}{27a^2}$

      iv. Deduce that the point of inflexion of $f(x)$ is at the midpoint of the stationary points.